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Hundreds of collisions between two hard needles

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Abstract. We found that two thin hard needles can collide hundreds of times without any additional forces. The number of collisions is sensitive to the initial conditions, especially the rotational phase of each needle. Long continuation of the chattering collision takes place at a limited region in the parameter space. The scattering angle was almost constant for the collision number in a frequent chattering collision, while the change in the collision number in a short chattering collision brings about a sizeable perturbation on the scattering process.

Two anisotropic hard bodies can collide more than once without any additional constraints, which is termed as chattering collision. The chattering collision is one of the difficult and open problems in the kinetic theory of gases [1], because it brings about extra correlation in the scattering process as demonstrated in the molecular dynamics simulation of model fluids [2]. We, however, know little of the characteristics of the chattering collision, because the collision dynamics of anisotropic hard bodies is governed by non-analytic equations. We carried out extensive numerical calculations on the collision dynamics between two thin hard needles, and found that the needles can collide more than a hundred times. A delicate tuning for the initial conditions is necessary to realize a long continuation of the chattering collision, because the collision number is very sensitive to the rotational phase of each needle.

Suppose an infinitely thin, homogeneous, and smooth hard needle as shown in figure 1. The needle has a unit mass and a unit length, and rotates around the unit vector $\boldsymbol{j}$ of the polar angles (θ, ϕ) with the angular velocity $\omega(>0)$. The direction of the needle is represented by the unit vector $\boldsymbol{u}$ and $\psi(t)(=\psi(0)+\omega t)$ denotes its rotational phase at time t. When the needle is on the xy-plane, $\psi(0)$ is set equal to zero.

We will consider the collision between the two hard needles A and B as described above. The origin is at the centre of mass of two needles. At $t=0$, the centres of two hard needles are on the xy-plane (0.5, $\pm b/2$, 0), where b is the impact parameter. Their initial translational velocities are $(\pm 0.5, 0, 0)$, which determine the unit time here.

When A and B collide at time t, the following equations hold [3]:

$$F(t)=(\boldsymbol{u}_{\mathrm{A}} \wedge \boldsymbol{u}_{\mathrm{B}}) \cdot (\boldsymbol{r}_{\mathrm{A}}-\boldsymbol{r}_{\mathrm{B}})=0 \qquad |\alpha_{\mathrm{A}}| \leqslant 0.5 \qquad |\alpha_{\mathrm{B}}| \leqslant 0.5 \tag{1}$$

where $\boldsymbol{r}_X$ is the position of the centre of the needle, X, and α_X is the distance of the contact position from the centre of the needle, X, measured along $\boldsymbol{u}_X$. Since the relative velocity between the contact positions of two needles at the collision instance is given by the time

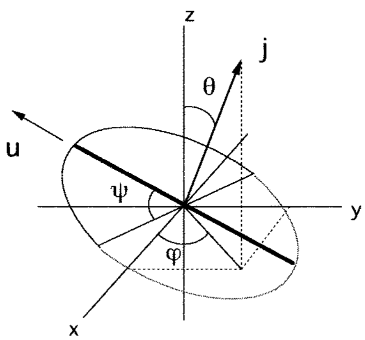

Figure 1. Model infinitely thin hard needle. The needle rotates around the axis $\boldsymbol{j}$.

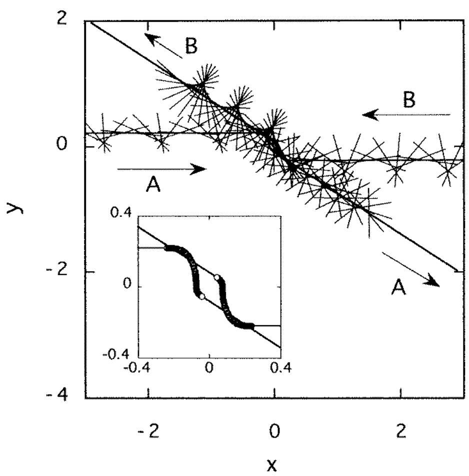

Figure 2. xy-projection of the trajectories of the needles, when they collide 71 times. The initial conditions are given in the text. Each short line represents snapshots of each needle with the time interval 0.3. Heavy lines are the trajectories of the centre of each needle. The inset shows the centres of the needles at each collision instance.

derivative of the minimum distance between two needles $|F(t)|/|\boldsymbol{u}_A \wedge \boldsymbol{u}_B|$, the amount of the momentum exchange $|\Delta P|$ is expressed as [3]

$$[1 + 6(\alpha_A^2 + \alpha_B^2)]|\Delta P| = \frac{1}{|\boldsymbol{u}_A \wedge \boldsymbol{u}_B|}\left|\frac{dF(t)}{dt}\right|. \tag{2}$$

When the needles slip on each other, the function $F(t)$ is constant at zero during the slip. Equation (2) implies that the two needles never slip on each other with a momentum transfer. The collision number, N, will not diverge, when a finite interaction acts at each collision instance.

If we know when (or where) the needles collide, the collision dynamics between the two needles is an elementary problem. However, it is unfeasible to solve equation (1) analytically, because the function $F(t)$ contains trigonometric functions in a complicated manner. We calculated numerically the trajectories of two hard needles by a modified technique in [3]. The major modification is an implementation of a bisection method together with the second-order Newton–Raphson method.

Numerical calculations show that the scattering process is very sensitive to the rotational phase, and that the chattering collision is rather common than rare when the impact parameter b is less than 0.5. Long continuation of the chattering collision ($N > 10$), however, was found in a very narrow range of $\psi_A(0)$ and $\psi_B(0)$. We will show the details of typical trajectories of two needles, in which the initial conditions are as follows. The impact parameter b is 0.44, the initial polar angles (θ, ϕ) of the rotational axes of A and B are $(0.71\pi, -0.57\pi)$ and $(0.18\pi, -0.41\pi)$, respectively, and their angular velocities ω_A and

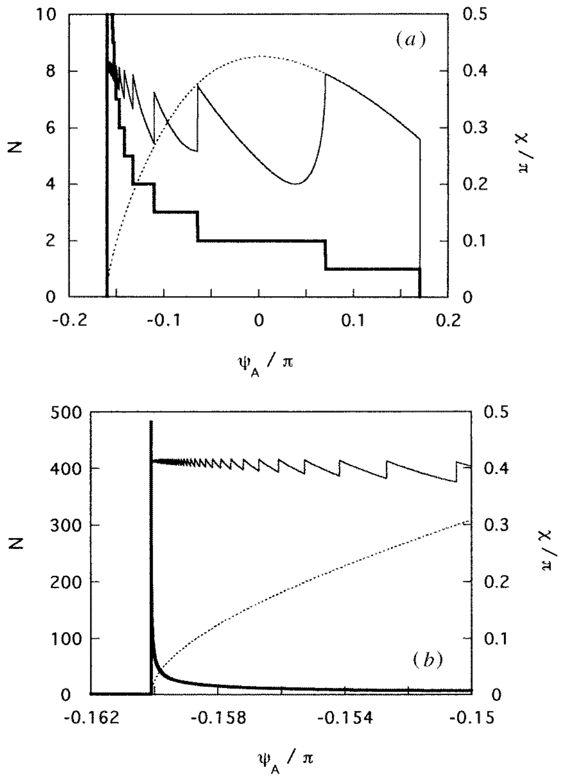

Figure 3. Dependence of the collision number N (heavy full curve) and the scattering angle χ (light full curve) on the initial rotational phase of the needle A, $\psi_A(0)$, at $\psi_B(0) = 0.17\pi$, in (a) the whole colliding region and (b) the region of the frequent chattering collision. The dotted curve represents the scattering angle at the first collision $\chi^{(1)}$.

ω_B are 1.71 and 1.92.

Figure 2 shows trajectories of the needles accompanying a frequent chattering collision ($N = 71$; $\psi_A(0) = -0.16\pi$, $\psi_B(0) = 0.17\pi$). The average time interval of the chattering collision is 0.0186 with the standard deviation 0.0050, and the centre of the needle moves by 0.523 during the chattering collision. The scattering angle χ of the overall process is 0.414π, and the average of the scattering angle in each collision is 0.0132π. The collision points on the needle A change gradually during the chattering collision from one end to the other. The first collision takes place at $\alpha_A = 0.488$ and $\alpha_B = 0.280$, and the last (71st) collision at $\alpha_A = -0.471$ and $\alpha_B = -0.471$.

Figure 3 shows the dependence of the number of collisions N and the scattering angle χ on the initial rotational phase of A, $\psi_A(0)$, at $\psi_B(0) = 0.17\pi$. The extent of $\psi_A(0)$ of a frequent chattering collision ($N > 10$) occupies 0.0059π in that of the chattering collision ($N > 1$) 0.23π (the extent of the whole colliding region amounts to 0.33π). The collision number is very sensitive to the initial conditions for the frequent chattering collision. When we decrease $\psi_A(0)$ from -0.1600π by $10^{-4}\pi$, the number of collisions increase from 71 to 401. However, no collision happens with the further decrease in $\psi_A(0)$ by $10^{-4}\pi$. The precision in the numerical calculation becomes a serious problem in the frequent chattering condition: for example the four-byte floating-point package gave 405 collisions at $\psi_A(0) = -0.1601\pi$ instead of 401 by the eight-byte or ten-byte floating-point package.

Decreasing $\psi_{\mathrm{A}}(0)$ from 0.170π, the scattering angle at the first collision $\chi^{(1)}$ is diminished with a maximum 0.462π around $\psi_{\mathrm{A}}(0) = 0$. The frequent chattering is accompanied with the diminution of $\chi^{(1)}$. The scattering angle of the whole scattering process χ changes suddenly as the collision number changes. The change in the scattering angle by the collision number is sudden but not always discontinuous; for example according to a detailed calculation, the change at the transition of $N = 1$–2 in figure 3(*a*) was continuous, while the changes at the transitions of $N = 0$–1 and $N = 2$–3 were discontinuous. The perturbation by each collision in the chattering collision decreases with increasing the collision number, and the collision number of the frequent chattering collision affects little the result of the whole scattering process. The scattering angle χ at $N = 401\,(0.4133\pi)$ is almost the same as that at $N = 71\,(0.4138\pi)$.

We found a long continuation of the chattering collision more than hundreds of times as described above. However, we have not known whether the collision number has an upper bound yet. We detected a trajectory with a very large number of collisions more than a million in a large set of the initial conditions, although two needles cannot slip on each other as shown in equation (2). The possibility of the infinite number of collisions in the chattering collision is still under study.

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